Motor Planning and Sparse Motor Command Representation*

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Abstract

The present article proposes a novel computational approach to the motor planning. In this approach, each motor command is represented as a linear combination of prefixed basis patterns, and the command for a given task is designed by minimizing a two-termed criterion consisting of a task optimization term and a parameter preference (i.e., sparseness) term. The result of a computer simulation with a single-joint reaching task confirmed that our "representation-based" criterion for motor planning appropriately worked, together with showing that the resultant trajectory qualitatively replicated Fitts' law.

keywords: *motor planning, sparse representation, parametric command representation*

1 Introduction

Although information representation in the brain is one of the central problems in neuroscience, most studies on motor control have not paid enough attention to this problem. The present article studies motor planning from a viewpoint of command representation.

Motor planning is a problem to design a command pattern for a given motor task, and many computational studies have been conducted to solve this problem. For the upper-limb reaching movement, for example, various optimality criteria have been proposed, including jerk-minimum principle[5] and torque-change minimum principle[11].

Whilst these theories succeeded in explaining the characteristic features of empirical data, they focused mainly on the kinematic/dynamic properties of movement trajectories, and did not try to design the command pattern directly. On the other hand, Harris & Wolpert[7] proposed a novel concept that the neural system designs a motor command whose resultant endpoint variance is minimized with a presence of signal dependent noise (SDN). This theory links the motor command and task requirement (i.e., endpoint accuracy) via SDN, and presents a method to plan motor commands based not on the kinematic space but on the command space.

In the present article, we aim to take another step toward studying the relationship between motor planning and command representation. In concrete, we propose a simple parametric model for command representation and a two-termed optimality criterion based on "sparseness" preference, referring to a computational theory on visual information representation. We formalize the problem and show how to solve it. Finally, we show the validity of the present framework by a computer simulation.

2 Parametric motor command representation

In our model, each motor command is represented as a linear combination of a fixed set of patterns, called "basis" patterns. This framework shares the idea with the synergy decomposition model[1, 2], where each motor command pattern to multiple muscles $\boldsymbol{u}(t)$ is represented as a linear sum of synergy patterns $\{\boldsymbol{w}_j(t)\}$ with weights $\{a_j\}$ and time shifts $\{d_j\}$ as

$$\boldsymbol{u}(t) = \sum_j a_j \boldsymbol{w}_j(t - d_j). \tag{1}$$

In the present study, we regard those patterns with different time shifts as different basis patterns, which results in

$$\boldsymbol{u}(t) = \sum_j a_j \boldsymbol{w}_j(t). \quad (a_j \geq 0) \tag{2}$$

In the following, we assume the basis is already given and we focus our discussion on how to find the optimal parameters $\{a_j\}$ for a given task, that is, motor planning[1].

3 Motor planning criterion dependent on command representation

One natural definition of the criterion for motor planning is the following:

$$E = [\text{Task Error}] + \lambda[\text{Additional Cost}]. \quad (\lambda > 0) \tag{3}$$

The first term corresponds to the task constraints/requirements, such as terminal condition (i.e., the hand must stay around the target for certain time), and is common to most computational models of motor planning. Because it is an "ill-posed problem" to find a solution satisfying simply the task requirement, various forms of additional costs have been proposed, such as jerk[5], torque-change[11], and endpoint variance[7], to determine the unique solution.

Here, we assume the preference on the parameters, instead of kinematics/dynamics cost, and use it as the second term. To this end, we refer to the computational

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[1] In the synergy decomposition model, the problem was to find optimal parameters and basis patterns which minimized the error between the observed muscle activities and reconstructed command patterns, which is different from motor planning.

model of visual information representation developed by Olshausen and Field[8].

In their scheme, a visual image $I(x, y)$ (x and y represents position in the image) was given by a generative model

$$I(x, y) = \sum_j a_j \phi_j(x, y) + \varepsilon(x, y), \qquad (4)$$

where $\{\phi_j(x, y)\}$ is the basis and $\varepsilon(x, y)$ is Gaussian noise. This framework has the same form as our parametric model in Eq. (2). They assumed that our visual system tried to minimize the following two-termed cost function, which is the logarithm of the posterior distribution,

$$\begin{aligned} &E(\{a_j\} \big| I; \{\phi_j\}) \\ &= \sum_{x,y} \left\| I(x, y) - \sum_j a_j \phi_j(x, y) \right\|^2 + \lambda \sum_j S(a_j) \\ &= [\text{Approximation Error}] + \lambda [\text{Parameter Preference}], \end{aligned} \qquad (5)$$

where $S(a_j)$ corresponds to the logarithm of the prior probability density function of a_j.

Notice that the second term of this cost function played an essential role in reproducing the receptive field properties of V1 neurons, and at the same time, gave rise to rich theoretical discussion, such as independent component analysis (ICA)[3] and sparse coding[9].

Comparing Eq.(3) with Eq.(5), we see that these cost functions well correspond to each other: The first terms represent the requirement of the system and the second terms represent the parameter preference. Considering this correspondence, we define a criterion for motor planning as follows

$$E = [\text{Task Error}] + \lambda \sum_j S(a_j). \qquad (6)$$

If we adopt $\sum_j |a_j|$ as the second term, that is, penalize larger weights, the system will tend to choose motor commands consisting of a smaller number of basis patterns. This means that the motor command is designed based on sparse representation.

4 Motor planning and optimization algorithms

In the proposed framework, the motor planning problem is solved by minimizing the cost function in Eq.(6). When the motor system is linear, the problem can be solved by well known optimization techniques by setting the [Task Error] term properly.

Let us imagine a reaching movement of a single-joint arm (See a detailed example in Sec. 5). If we define the [Task Error] term as $\sum_{t=t_f}^{t_f+t_p} |\theta(t) - \theta_T|^2$, and the second term as $\sum_j |a_j|^p$, then the problem becomes

$$\text{minimize} \sum_{t=t_f}^{t_f+t_p} |\theta(t) - \theta_T|^2 + \lambda \sum_j |a_j|^p, \qquad (7)$$

where $\theta(t)$ is the joint angle at time t, θ_T is the target. We consider t as discrete time step, and t_f and t_p are movement time and pausing time, respectively. We usually set $p = 1$ (i.e., L_1 constraint) for sparseness perference, but below we also think of the case $p = 2$ (i.e., L_2 constraint), which prefers the minimum "energy" solution, for comparison.

Because $\theta(t)$ can be written as a linear combination of $\{a_j\}$, this problem can be solved using the quadratic programming (QP) for $p = 1$ and 2. Note that this problem is similar to the "Lasso" model[10] when $p = 1$.

On the other hand, we can define $l_e(\cdot)$ as

$$l_e(x) = \begin{cases} 0 & |x| \le e \\ \infty & |x| > e \end{cases}, \quad (e \ge 0), \qquad (8)$$

and define the [Task Error] term as $\sum_{t=t_f}^{t_f+t_p} l_e(\theta(t) - \theta_T)$, which forces $\theta(t)$ to stay between $\theta_T \pm e$ during $[t_f, t_f + t_p]$ and [Task Error] $= 0$. In this case, the problem can be rewritten as

$$\text{minimize} \sum_j |a_j|^p \;\text{ s.t. }\; |\theta(t) - \theta_T| \le e \quad (t_f \le t \le t_f + t_p). \qquad (9)$$

This can be solved with the linear programming (LP) and QP when $p = 1$ and $p = 2$, respectively.

Application of these algorithms dramatically reduces the computational cost, and therefore, the motor planning can be solved quite efficiently.

5 Simulated experiment

5.1 Method

In order to examine the validity of above framework and the effect of different optimization methods, we ran a computer simulated experiment, taking a single-joint reaching task as an example.

Figure 1 (a) shows the structure of the model, which is almost the same as one-dimensional arm model in Harris & Wolpert[7]. The motor command $u(t)$ was transferred into the joint torque $\tau(t)$ through two first-order low-pass filters, whose time constants (T_1 and T_2) were 30 and 40 ms, respectively:

$$(T_1 T_2) \frac{d^2\tau}{dt^2}(t) + (T_1 + T_2) \frac{d\tau}{dt}(t) + \tau(t) = u(t). \qquad (10)$$

Dynamics of the arm was given by

$$I \frac{d^2\theta}{dt^2}(t) + b \frac{d\theta}{dt}(t) + k l_0^2 \theta(t) = \tau(t), \qquad (11)$$

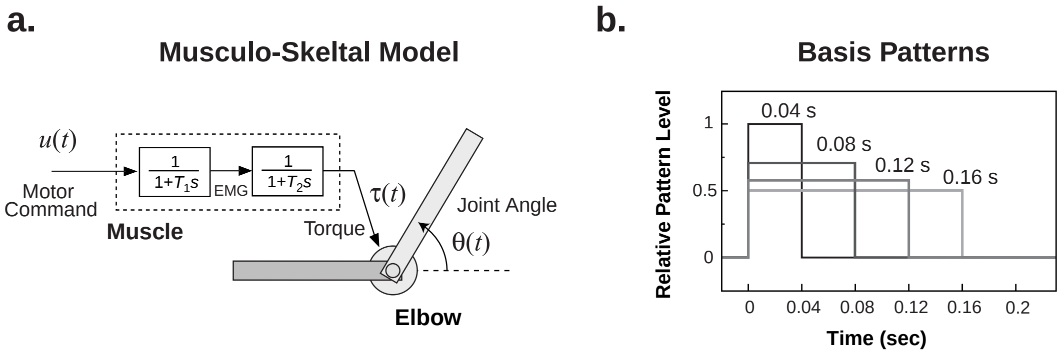

Figure 1. Model structure and pre-defined basis patterns. **a.** Motor command is imposed to the single-joint skeletal system through two first-order linear filters. **b.** Four pre-defined basis patterns are prepared.

where $\theta(t)$ is the joint angle, $I(=0.25)$ is the inertia, $b(=0.2)$ is the viscosity, $k(=0.1)$ is the elasticity, $l_0(=0.35)$ is the arm length. In total, therefore, the motor system was a fourth order linear system.

On the other hand, the motor command $u(t)$ was given by Eq.(2). In this experiment, the pre-defined basis patterns $w_j(t)$ were four rectangular pulses with different time-width but with the same energy (i.e., $\sum_t w_j^2(t)\mathit{\Delta}_t = 1$, where $\mathit{\Delta}_t$ is the width of discrete time step. See Figure 1 (b)). This assumption is based on the discussion that single joint movement can be achieved by controlling the widths and heights of rectangular motor commands [6]. To be more specific, we prepared positive and negative rectangular pulses starting at 30 different times (from 0 s to 0.8 s with an interval of 0.04 s).

Task performance was evaluated by the sum of the endpoint error during the pausing time $t_p(=0.4$ s) and movement time t_f with a weight η (i.e., faster movement was preferable). Resultantly, our objective function was

$$E(\{a_j\}, t_f \big| \theta_T; \{w_j\}) = \eta\, t_f + [\text{Task Error}] + \lambda \sum_j |a_j|^p. \tag{12}$$

Note that movement time was also determined by optimizing this function.

We solved this problem using LP and QP by rewriting it as in Eqs.(7) and (9). In concrete, we solved the problem using these algorithms for each t_f value chosen from the region [0.1 s, 0.4 s] with an interval of 0.02 s and then searched the optimal value of t_f which gave the minimal cost.

Other experimental conditions were as follows: The initial joint angle was 0 rad, and the target angle (θ_T) was chosen from 0.2, 0.4, 0.8 and 1.6 rad. ($\simeq \pi/2$). Optimization problem was solved with IMSL C Math Library (Visual Numerics, Inc.). We ran the simulation for different values of parameters λ, η and e, and found their appropriate values producing meaningful results.

5.2 Result

Figure 2 (a) shows the command patterns and joint trajectories obtained by four different optimization methods. Parameter values were chosen so that the resultant movement times were similar for all methods. Typical three-phase command patterns and smooth movement trajectories were generated, and angular velocity (data not shown) showed an almost-symmetric bell-shaped profile, common to all conditions. This confirms that the proposed method worked appropriately as a motor planning algorithm.

Two other major features are pointed out here. First, while the movement trajectories were similar among different optimization formalizations, the command patterns showed different features: Command patterns for $p = 2$ (B and D in Fig. 2(a)) were more continuous and minute (i.e., containing a larger number of basis patterns), compared to those for $p = 1$ (A and C). This means that sparser representation were obtained with L_1 constraints on the weights.

Second, movement time t_f depended on θ_T. Figure 2 (b) shows this dependency in more detail[2]. The upper panel indicates the joint trajectories for different values of η when we solve Eq.(7) with $p = 1$, and the lower panel shows the relationship between the log movement distance and movement time for different η values. The data for four targets are nicely located on the regression line. This means that the present result reproduced Fitts' law[4], saying that the movement time can be approximated by $a + b$ (log(l) - log(w)), where l is movement distance and w is target size (a and b are constants).

6 Concluding remarks

In the present article, we introduced a parametric framework of motor command representation, and showed the possibility of the parameter preference to work as a criterion for motor planning.

[2]This dependency changed according to η when λ and e were fixed. Meaningful results were obtained for appropriate values of η: t_f was stuck to the upper or lower limit for too small or too large η, respectively.

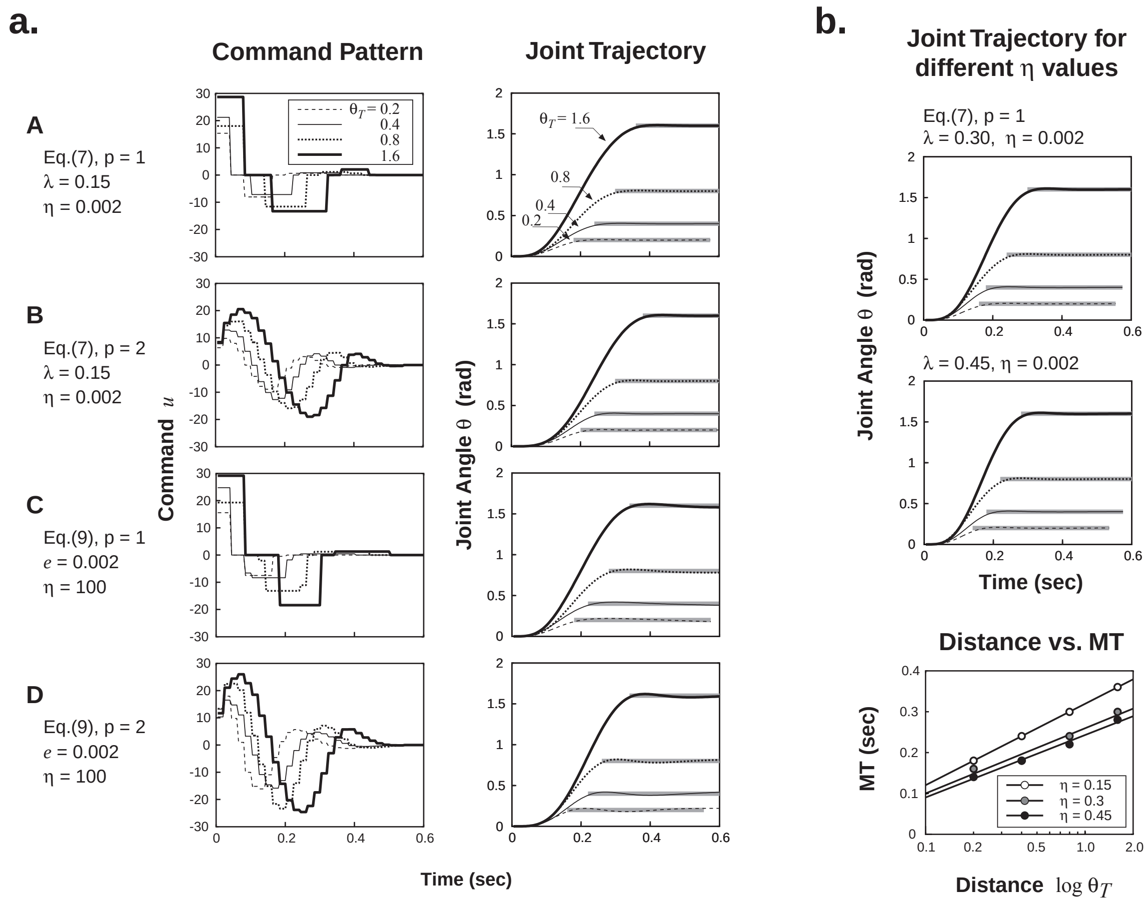

Figure 2. Results of computer simulated experiments. **a.** Planned command patterns and resultant trajectories for four different optimization methods: (A) QP solution to Eq.(7) with $p = 1$, (B) QP solution to Eq.(7) with $p = 2$, (C) LP solution to Eq.(9) with $p = 1$, and (D) QP solution to Eq.(9) with $p = 2$. **b.** Relationship between log movement distance and movement time for different η values in case A.

A simulated experiment shows that Fitts' law can be qualitatively replicated with a single-joint linear arm model. We are now examining whether our representation-based criterion also works for the movement with non-linear and/or redundant motor systems. Creating optimal basis for a given task environment is a future research topic.

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