Joint 48th IEEE Conference on Decision and Control and
28th Chinese Control Conference
Shanghai, P.R. China, December 16-18, 2009

Motor Planning as an Optimization of Command Representation

Shiro Ikeda and Yutaka Sakaguchi

***Abstract*— A fundamental problem in the field of motor neuroscience is to understand how our brain generates appropriate motor commands for precise movements effortlessly. The problem seems difficult since there are infinitely many possible trajectories and our musculo-skeltal system is generally redundant. We focus on the motor command representation and show that a simple strategy can solve the problem for a planar two-joints arm model. We also discuss the emergence of the muscle synergies, which may enable us to make natural motor behaviors with smaller degrees of freedom.**

I. INTRODUCTION

In daily life, humans can effortlessly control an arm to reach a target. Although there are infinitely many candidates of paths and velocity profiles which achieve the task, typical hand paths are gently curved and velocity profiles are bell-shaped. Also, our muscle system to drive the arm is redundant. One of the fundamental problems of motor neuroscience is to understand how our brain generates a set of appropriate motor commands to achieve the task.

In 1980's, it was shown that criteria based on physical quantities, such as the minimum jerk [1] and the minimum changes in joint torques [2], explain hand paths and velocity profiles well. In 1998, Harris and Wolpert reported that the assumption of signal dependent noise describes many characteristics of the motor control [3]. Recently, Haruno and Wolpert showed that the signal dependent noise gives a clue to solve the muscle redundancy [4].

In this paper, we solve the problem from a different viewpoint. In order to move an arm to a target, there are infinitely many possible trajectories. Moreover every trajectory can be realized with multiple motor commands because of the redundant muscle system. Therefore, choosing a single motor command, which we call "motor planning," is the planning not only of a trajectory, but also of a particular activation pattern of muscles. In order to solve the problem, we do not consider any physical quantities, such as jerk or torque change, nor noise, but assume the functional form of motor commands with some parameters. Each motor command is uniquely represented with the parameters. We call it "motor command representation," and define a cost function which describes a preference of the representation. We propose a strategy to choose a single motor command based on the preference (Sec.II). The new proposal is examined with reaching tasks (Sec.IV) of a two-joints arm (Sec.III). We show the resulting motor commands possess typical characteristics of the human reaching movements through simulated experiments (Sec.V).

We further discuss the emergence of synergies. The muscle synergies are defined as the coherent activations of a group of muscles [7]. We show that the motor commands obtained through our strategy form groups of motor commands which are similar to muscle synergies (Sec.V-C). Finally the paper is concluded with a short discussion (Sec.VI).

This work was supported by the Grant-in-Aid for Scientific Research No. 18079013 to SI and No. 21300092 to YS, MEXT, Japan.

S. Ikeda is with Department of Mathematical Analysis and Statistical Inference, The Institute of Statistical Mathematics, 10-3 Midoricho, Tachikawa-shi, Tokyo, 190-8562, Japan. shiro@ism.ac.jp

Y. Sakaguchi is with Graduate School of Information Systems, University of Electro-Communications, 1-5-1 Chofugaoka, Chofu-shi, Tokyo, 182-8585, Japan. sakaguchi@is.uec.ac.jp

II. OPTIMIZING MOTOR COMMAND REPRESENTATION

Every motor command is a time series sent from brain to muscles. Here, we pose a question, "how can brain create and send time series?" The answer to this question has not been made clear in the motor neuroscience.

In this paper, we assume the functional representation of the motor command as a linear combination of a prefixed basis, more precisely as follows

$$u_i(t) = \sum_j w_{ij}\phi_j(t), \quad w_{ij} \geq 0,$$

where u_i is the motor command to muscle i and $\{\phi_j\}$ is the basis. Since $W{=}(w_{ij})$ defines the motor command, we call it a "motor command representation." The basis is defined as a set of synchronizing patterns with different durations, which is similar to what is discussed in [8].

Now, our problem is to select a single motor command representation W. We solve this problem by assuming sparsity. Olshausen and Field assumed sparsity on the visual representation and discussed the optimal basis, which possess the characteristics of the simple cells observed in the primary visual cortex [9]. Here, we assume the basis and compute the optimal representation based on sparsity. We define the following cost function

$$\mathrm{Preference}(W;\lambda_1,\lambda_2) = \lambda_1 \sum_{ij} w_{ij} + \lambda_2 \sum_{ij} w_{ij}^2, \quad \text{where} \quad \lambda_1, \lambda_2 > 0.$$

We obtain our "preferred representation" by minimizing this cost function. The weight of the first term λ_1 is important for the sparsity, while the weight of the second term controls the smoothness of the representation. A similar idea was proposed in [10]. We choose appropriate values of these parameters through some trials.

978-1-4244-3872-3/09/$25.00 ©2009 IEEE

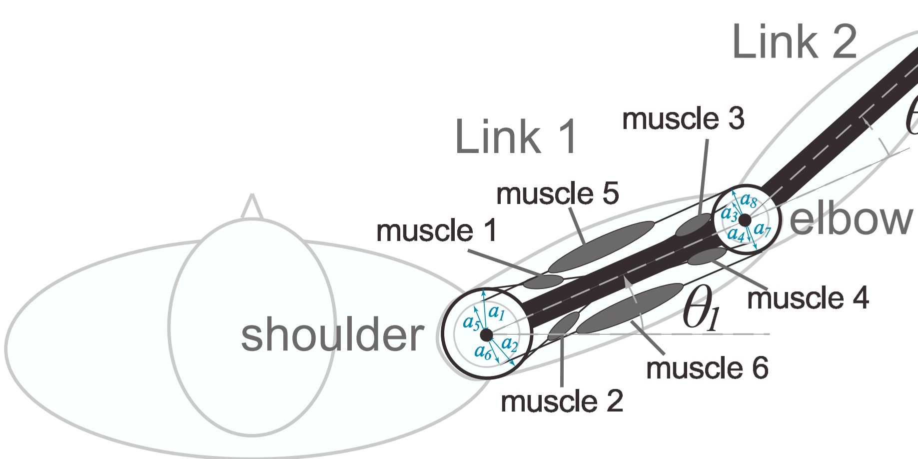


Fig. 1. Two joints arm model with six muscles. The upper arm is "Link 1" (length l_1 is 0.275 m and the moment of inertia I_1 is 0.029 kgm^2). The forearm is "Link 2" ($l_2 = 0.345$ m, $I_2 = 0.042$ kgm^2, and the weight m_2 is 1.077 kg). The upper arm is connect to the shoulder and the forearm via two joints. The torques of two joints are produced by 6 muscles. Each muscle is connected to each joint as in the figure where $(a_1, \cdots, a_8)$ = (3.5 cm, 4.1 cm, 2.7 cm, 2.0 cm, 2.9 cm, 4.3 cm, 2.5 cm, 2.3 cm). The parameters are the same as those in [13].

We give a simple example. Suppose u is the input of the following linear forward dynamics of x,

$$\dot{x} = a\ x + b\ u(t) = a\ x + b\ \sum_j w_j \phi_j(t),$$

Assuming $\Phi_j(t)$ is the response of the system when $u = \phi_j(t)$, then from the linearity of the system,

$$x(t) = \sum_j w_j \Phi_j(t).$$

When the achievement of a task is evaluated with a function $\mathrm{Error}(x(t))$ of $x(t)$, such as the endpoint error, our proposal is to select the $\{w_j\}$ which minimizes

$$\begin{aligned}\mathrm{Cost} &= \mathrm{Error}(x(t)) + \mathrm{Preference}(W; \lambda_1, \lambda_2)\\ &= \mathrm{Error}(W : \{\Phi_j(t)\}) + \mathrm{Preference}(W; \lambda_1, \lambda_2).\end{aligned}$$

If $\mathrm{Error}(x(t))$ is a linear or quadratic function of $x(t)$, Cost becomes a quadratic function of $\{w_j\}$, and optimal W is solved with a quadratic programming (QP) method.

In the following, we use a two-joints arm, which is a nonlinear system, to show how the proposed strategy works.

III. MODEL

A. Two-joints Arm

In this paper, we consider a 2-joints (shoulder and elbow) 6-muscle arm (Fig.1) [11], [12], [13]. The inverse dynamics of the arm in the horizontal plane is

$$M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + B\boldsymbol{\theta} = \boldsymbol{\tau}(t). \qquad (1)$$

The forward dynamics becomes

$$\ddot{\boldsymbol{\theta}} = -M(\boldsymbol{\theta})^{-1}\big(C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + B\boldsymbol{\theta} - \boldsymbol{\tau}(t)\big),$$

where $\boldsymbol{\theta}(t) = (\theta_1(t), \theta_2(t))^T \in \Re^2$ is the angle vector (θ_1: shoulder, θ_2: elbow), and $\boldsymbol{\tau}(t) = (\tau_1(t), \tau_2(t))^T \in \Re^2$ is the torque vector, (τ_1: shoulder, τ_2: elbow). The parameters are given as

$$M(\boldsymbol{\theta}) = \begin{pmatrix} \alpha_1 + 2\alpha_2\cos\theta_2 & \alpha_3 + \alpha_2\cos\theta_2 \\ \alpha_3 + \alpha_2\cos\theta_2 & \alpha_3 \end{pmatrix}$$

$$C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \begin{pmatrix} -\dot{\theta}_2(2\dot{\theta}_1 + \dot{\theta}) \\ \alpha_3 + \alpha_2\cos\theta_2 \end{pmatrix}\alpha_2\sin\theta_2,\ \ B = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix}$$

$$\alpha_1 = I_1 + I_2 + m_2 l_1^2, \quad \alpha_2 = m_2 l_1 s_2, \quad \alpha_3 = I_2,$$

where, I_i is the moment of inertia of each link, m_2 is the weight of link 2, l_i is the length of link i, s_2 is the distance from the joint center to the center of the mass of link 2 (0.162 cm), and $\beta_{11} = 1.445$, $\beta_{12} = \beta_{21} = 0.301$, $\beta_{22} = 1.383$. The parameters are the same as those in [13]. The forward dynamics is computed with the modified Euler method, where the updating interval is 5 msec.

B. Torque and Moment Arm

A set of motor commands, $\boldsymbol{u}(t) = (u_1(t), \cdots, u_6(t))^T$, ($u_i(t) \geq 0$) activates 6 muscles and their tensions are combined to give the two dimensional torque $\boldsymbol{\tau}(t)$. Figure 2 schematically shows the model of the process. We assumed $\boldsymbol{u}(t)$ is processed through 2 low-pass filters (1st-order low-pass filters with $T_1 = 30$ msec and $T_2 = 40$ msec. These values were chosen according to [3]). We assume the tension of each muscle is proportional to the output of low-pass filters, which we define $\boldsymbol{u}'(t)$. Since each muscle has a different strength depending on the cross-sectional area, unit-less $\boldsymbol{u}'(t)$ is correctly scaled to give the tension $\boldsymbol{T}(t)$ as

$$\boldsymbol{T}(t) = D\boldsymbol{u}'(t),$$

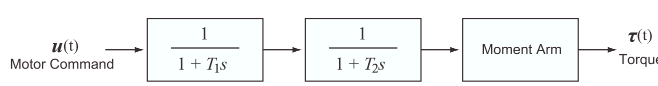


Fig. 2. The process from motor command to torque.

where D is a diagonal matrix with $\mathrm{diag}(d_1, \cdots, d_6) = (840\ \mathrm{N}, 800\ \mathrm{N}, 560\ \mathrm{N}, 480\ \mathrm{N}, 200\ \mathrm{N}, 240\ \mathrm{N})$. The diagonal elements correspond to the maximum tension of the muscles, computed by the cross-sectional area of each muscle [14] and the maximum tension per unit area (we set it to 62 $\mathrm{N/cm^2}$). Finally $\boldsymbol{T}(t)$ is multiplied by the moment arm (Fig.1) to give the torque as

$$\boldsymbol{\tau}(t) = A\ \boldsymbol{T}(t) = A\ D\ \boldsymbol{u}'(t),$$

where

$$A = \begin{pmatrix} a_1 & -a_2 & 0 & 0 & a_5 & -a_6 \\ 0 & 0 & a_3 & -a_4 & a_7 & -a_8 \end{pmatrix}.$$

We assumed a constant moment arm, that is, A was fixed.

C. Motor Command

As described in section II, we assume that the motor command is represented as a linear combination of a basis, which is a set of pre-fixed time series. In this paper, we defined a basis with 3 kinds of single-shot positive square wave (with length of 0.05, 0.1, and 0.2 sec) shown in Fig.3(a). Let us denote them as $\phi_i(t)$, $i = 1, 2,$ and 3. We assume the basis is created with a synchronous timing T_s sec generated by the brain. In our experiment, T_s is set to 0.3 sec. Thus, a motor command $u_i(t)$ is written as

$$u_i(t) = \sum_{k=0}^{K}\sum_{j=1}^{3} w_{ijk}\phi_j(t - kT_s), \quad w_{ijk} \geq 0. \tag{2}$$

The output of the low-pass filters $u_i'(t)$ is written with a linear mixture of $\{\phi_i'(t)\}$ which are the low-passed version of $\{\phi_i(t)\}$. Figure 3(b) shows the function,

$$u_i'(t) = \sum_{k=0}^{K}\sum_{j=1}^{3} w_{ijk}\phi_j'(t - kT_s).$$

Note that $u_i(t), u_i'(t) \geq 0$ from the definition.

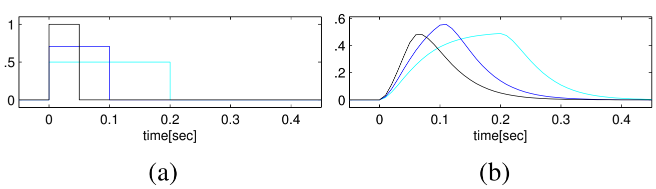


Fig. 3. Basis of the motor command: (a) a set of function for motor command $\phi_i(t)$, (b) the outputs of each function through low-pass filters $\phi_i'(t)$.

Finally, the motor command $\boldsymbol{u}(t)$ becomes a function of $\{w_{ijk}\}$. Let us denote $\{w_{ijk}\}$ with $\boldsymbol{w}$, and motor command is $\boldsymbol{u}(t; \boldsymbol{w})$. We also note that $\boldsymbol{\theta}(t)$ is a function of $\boldsymbol{w}$, that is, $\boldsymbol{\theta}(t; \boldsymbol{w})$. The motor planning is to compute the $\boldsymbol{w}$ which achieves the given task of $\boldsymbol{\theta}(t; \boldsymbol{w})$.

IV. OPTIMAL MOTOR COMMAND FOR REACHING TASK

A. Reaching Task and Cost Function

The task of reaching is to move the hand from an initial position $\boldsymbol{\theta}_I$ to a target position $\boldsymbol{\theta}_T$. We evaluate the achievement of the task with the endpoint error.

$$\mathrm{Error}(\boldsymbol{w}) = \frac{1}{T_f}\int_{T_e}^{T_e+T_f} |\boldsymbol{\theta}(t; \boldsymbol{w}) - \boldsymbol{\theta}_T|^2 dt,$$

where T_e is the desired movement time and T_f is the post-movement stationary time. Since we are working with a discrete time, Error is redefined as

$$\mathrm{Error}(\boldsymbol{w}) = \frac{1}{T_f(L+1)}\sum_{l=0}^{L} |\boldsymbol{\theta}(t_l; \boldsymbol{w}) - \boldsymbol{\theta}_T|^2, \quad \text{where} \quad t_l = T_e + l\frac{T_f}{L}. \tag{3}$$

The motor command is represented by $\boldsymbol{w}$, and the command which achieves the reaching task can be computed by minimizing $\mathrm{Error}(\boldsymbol{w})$. However, it does not give a unique $\boldsymbol{w}$ since the muscles are redundant.

As is discussed in the section II, we further assume a preference on the parameters $\boldsymbol{w}$, which is defined as

$$\mathrm{Preference}(\boldsymbol{w}; \lambda_1, \lambda_2) = \lambda_1 \sum_{ijk} w_{ijk} + \lambda_2 \sum_{ijk} w_{ijk}^2.$$

Note that the second term shows the power of $\boldsymbol{w}$ while the first term adds the sparsity to $\boldsymbol{w}$.

Therefore, the cost function we have defined in Sec.II is written as follows,

$$\begin{aligned}\mathrm{Cost}(\boldsymbol{w}; \lambda_1, \lambda_2) &= \mathrm{Error}(\boldsymbol{w}) + \mathrm{Preference}(\boldsymbol{w}; \lambda_1, \lambda_2) \\ &= \frac{1}{T_f(L+1)}\sum_{l=0}^{L} |\boldsymbol{\theta}(t_l; \boldsymbol{w}) - \boldsymbol{\theta}_T|^2 \\ &\quad + \lambda_1 \sum_{ijk} w_{ijk} + \lambda_2 \sum_{ijk} w_{ijk}^2\end{aligned}$$

The optimal $\boldsymbol{w}$, which minimizes the cost function, gives the motor command for the given reaching task.

$$\hat{\boldsymbol{w}} = \underset{\boldsymbol{w}}{\mathrm{argmin}}\Big[\mathrm{Cost}(\boldsymbol{w}; \lambda_1, \lambda_2)\Big]. \tag{4}$$

B. Minimizing Cost Function

Since $\boldsymbol{\theta}(t; \boldsymbol{w})$ is a nonlinear function of $\boldsymbol{w}$, it is difficult to solve eq.(4) analytically. We locally approximate $\boldsymbol{\theta}(t; \boldsymbol{w})$ with

$$\boldsymbol{\theta}(t; \boldsymbol{w}_{(ijk)} + \Delta) \simeq \boldsymbol{\theta}(t; \boldsymbol{w}) + \Delta\frac{d\boldsymbol{\theta}(t; \boldsymbol{w})}{dw_{ijk}},$$

where $\boldsymbol{w}_{(ijk)} + \Delta$ denotes that Δ is added to w_{ijk}. We define Δ_{ijk} as the solution of the following optimization problem.

$$\begin{aligned}\min\ &\left[\frac{1}{T_f(L+1)}\sum_{l=0}^{L}\left|\sum_{ijk}\Delta_{ijk}\frac{d\boldsymbol{\theta}(t_l; \boldsymbol{w})}{dw_{ijk}} + \boldsymbol{\theta}(t_l; \boldsymbol{w}) - \boldsymbol{\theta}_T\right|^2\right. \\ &\left. + \lambda_1 \sum_{ijk}(w_{ijk} + \Delta_{ijk}) + \lambda_2 \sum_{ijk}(w_{ijk} + \Delta_{ijk})^2\right], \\ &\text{subject to} \quad \Delta_{ijk} \geq -w_{ijk}.\end{aligned}$$

This problem is easily solved with a QP method. The derivative $d\boldsymbol{\theta}(t; \boldsymbol{w})/dw_{ijk}$ is approximated by adding a small positive perturbation δ to w_{ijk} and computing the resulting differential equation of $\boldsymbol{\theta}$, that is

$$\frac{d\boldsymbol{\theta}(t;\boldsymbol{w})}{dw_{ijk}} \simeq \frac{1}{\delta}(\boldsymbol{\theta}(t;\boldsymbol{w}_{(ijk)}+\delta)-\boldsymbol{\theta}(t;\boldsymbol{w})).$$

After optimizing Δ_{ijk}, every w_{ijk} is renewed as $w_{ijk}+\Delta_{ijk}$, and the process is iterated until convergence. If the model is a linear control system, the state of the system becomes linear w.r.t. $\{w_{ijk}\}$ and no iteration is needed. From the experiment, we see it converges surprisingly well after 3 or 4 iterations. It implies the control system in eq. (1) is not strongly nonlinear.

V. EXPERIMENT

A. Task Set

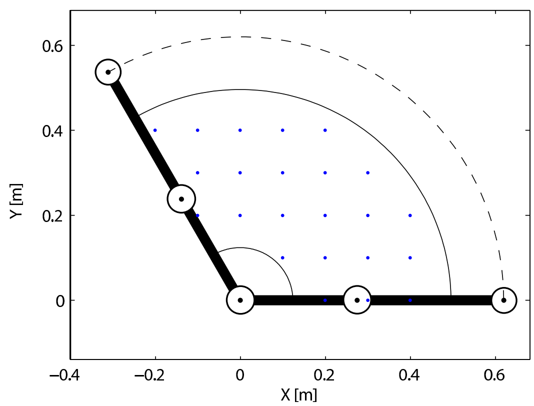


Fig. 4. Candidates of the initial and terminal points.

We computed the optimal motor commands for a set of initial-target position pairs (Fig.4 shows the points which are used as initial and target positions). We first set a fan shape region, the range of $\boldsymbol{\theta}$ is $0 \le \theta_1 \le 2\pi/3$, $\theta_2 = 0$ and distance from the origin is between $0.2l_a$ to $0.8l_a$, where $l_a = l_1 + l_2$. We chose grids every 10 cm on the horizontal plane, and if the grid falls into the fan shape region, it is a candidate of the initial and terminal points.

Every pair of points is chosen if the distance between them is more than or equal to 20 cm. There are 380 pairs which satisfies the condition, and we used all of them. This is the task set.

The values L, T_e, and T_f in eq.(3) are set to 8, 0.4 sec, and 0.4 sec, respectively, and λ_1 and λ_2 in eq.(4) are set to 2×10^{-6} and 1×10^{-5}, respectively[1].

B. Results

Out of 380 reaching tasks, 4 results are summarized in Fig.5. The results in Fig.5(a) show slightly curved trajectories, which are the typical characteristics of reaching tasks. The velocity profiles in Fig.5(b) clearly form bell-shapes, which are also typically observed in real experiments.

Figure 5(c) shows the low-passed motor commands of 6 muscles. Many of the motor commands become 0, that is, the motor commands are sparse, and the redundancy of muscles are clearly removed.

C. Synergies

Furthermore, we observe some groups of muscles tend to be activated simultaneously. This observation motivates us to give further analysis. The idea of muscle synergies has been discussed in [7]. The muscle synergies are the "*coherent activations, in space or time, of a group of muscles*," which is considered to be "*building blocks that could simplify the construction of motor behaviors*," (from [7]). In [7], the NMF-type (Nonnegative Matrix Factorization) analysis is applied to the measured EMG. In our case, we apply NMF-type approach to the coefficients of the motor commands, and see if we observe some interesting results.

The motor command $\boldsymbol{u}(t)$ is represented by eq. (2). We first define the normalized coefficient vector $\boldsymbol{\omega}_k$ as follows,

$$\boldsymbol{\omega}_k = \frac{1}{|(w_{11k}, w_{12k}, \cdots, w_{63k})|}(w_{11k}, w_{12k}, \cdots, w_{63k})^T.$$

$\boldsymbol{\omega}_k$ is a vector of each time step and its dimension is (# of basis functions × # of muscles) $= 3\times6 = 18$. Non-zero $\boldsymbol{\omega}_k$ are collected from the results of 380 reaching tasks, and renumbered to form a matrix W as

$$W = (\boldsymbol{\omega}_1, \cdots, \boldsymbol{\omega}_N).$$

In our case, $N = 1140 (= 3 \times 380)$, which shows each of 380 tasks has 3 non-zero $\boldsymbol{\omega}_k$, and W is a matrix with a size of 18×1140, where every component is positive and the squared length of each column is 1.

We define synergy vectors $\boldsymbol{s}_1, \cdots, \boldsymbol{s}_M$, $M < 18$, where $\boldsymbol{s}_m$ is an 18 dimensional vector with positive components. Let us define a synergy matrix $S = (\boldsymbol{s}_1, \cdots, \boldsymbol{s}_M)$, and assume

$$W \simeq SH, \quad \text{where} \quad H = (h_{mn}) \in \Re_+^{M\times N}.$$

We would like to compute S and H when W is given. This is the NMF problem [15]. We solved the following problem

$$\min\Big[\ \|W - SH\|_F^2 + \lambda\sum_{i,m} s_{im}\Big],$$
$$\text{subject to} \quad s_{im} \ge 0,\ h_{mn} \ge 0,$$

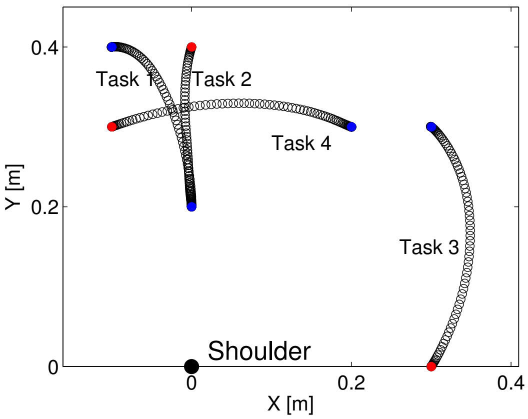
where $\|\cdot\|_F$ is the Frobenius norm and the second term makes S sparse (λ is set to 1×10^{-5}). The problem is easily solved with a QP method. We applied a QP method to compute H and S iteratively. Although the algorithm has the initial condition dependence, it monotonically converges to a local minimum. We varied M from 4 to 8, and computed the synergies with different initial values. It is easy to imagine that as the number of the synergies increases, $\|W - SH\|_F^2$ becomes smaller, but the synergies become isolated commands on each muscle. Figure 6 shows the low-passed outputs of synergies, when M is set to 5.

With this set of synergies $\hat{S}$, W is reconstructed as

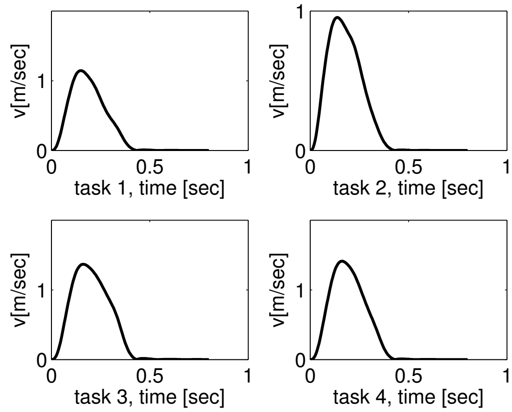
$$\hat{\boldsymbol{\omega}}_n = \hat{S}\hat{\boldsymbol{h}}_n,$$

where $\hat{H} = (\hat{\boldsymbol{h}}_1, \cdots, \hat{\boldsymbol{h}}_N)$. The average squared error $\sum_{n=1}^{N}|\boldsymbol{\omega}_n - \hat{\boldsymbol{\omega}}_n|^2/N$ is 5.1%.

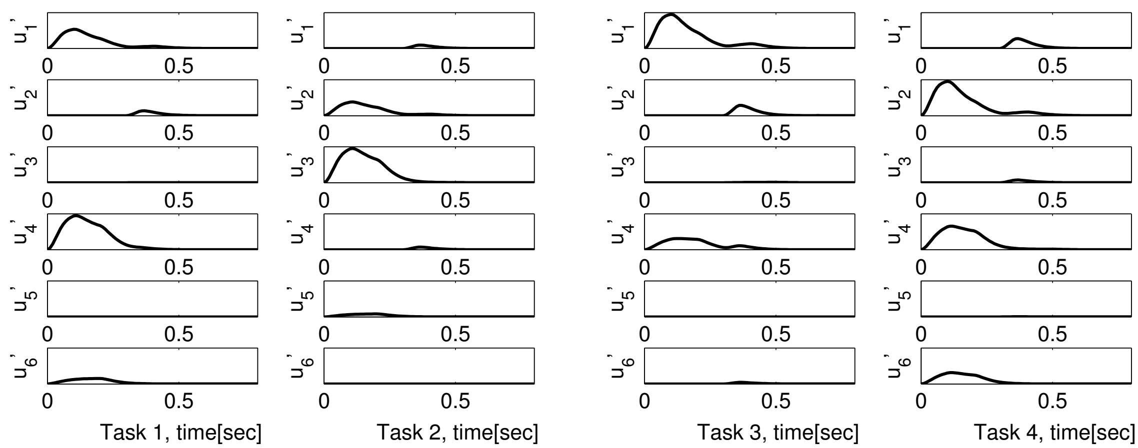
[1] A lot of combinations of (λ_1, λ_2) are tested. The values are chosen to make the resulting movement smooth and the motor command representation sparse.



(a) Trajectories



(b) Velocity profiles



(c) Low-passed motor commands of 6 muscles

Fig. 5. Results: 4 reaching tasks out of 380 are shown. Each task is as follows (denoted with x-y coordinate (x cm, y cm)) : Task 1; from $(0, 20)$ to $(-10, 40)$, Task 2; from $(0, 40)$ to $(0, 20)$, Task 3; from $(30, 0)$ to $(30, 30)$, and Task 4; from $(-10, 30)$ to $(20, 30)$, (a) shows the trajectories, where red and blue dots are initial and terminal points, respectively, (b) the velocity profile of each task, (c) the low-passed motor commands of 6 muscles.

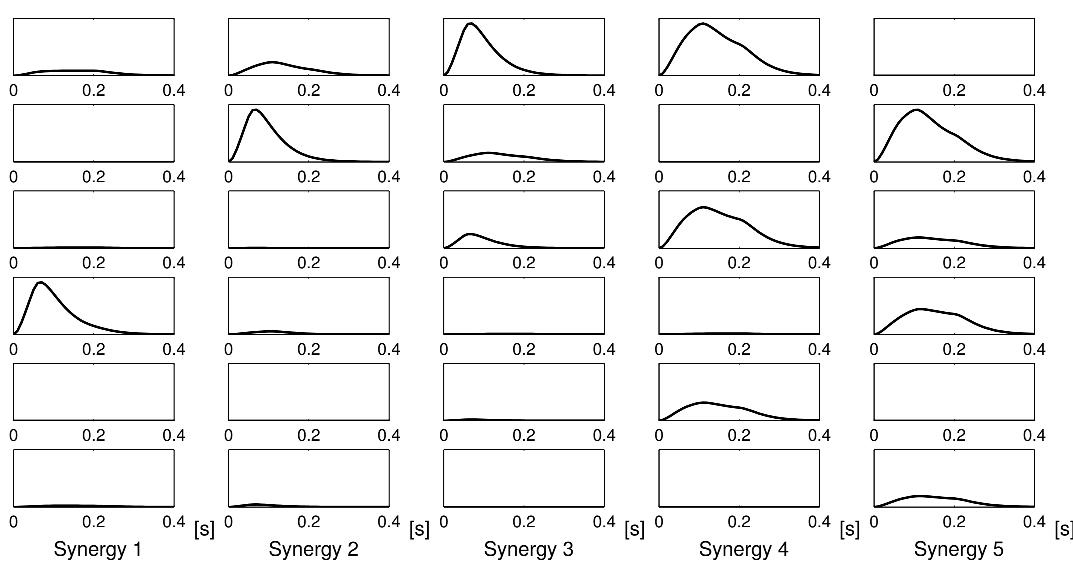


Fig. 6. Synergies.

D. Arbitrary Initial and Target Positions

It is difficult to imagine that brain is optimizing the motor command representation for every motor planning. A more plausible idea is that brain stores learned motor commands and somehow creates a mapping function from task to motor command based on the learned motor commands. We consider such an extension of our proposal.

In our model, the motor command is represented with a finite number of positive coefficients, and the mapping function should have a form

$$\boldsymbol{w} = f(\boldsymbol{\theta}_I, \boldsymbol{\theta}_T).$$

We have already computed the motor commands for grids (Fig. 4). Let a or b denote the index of each grid, where the initial and target points are indexed as θ_I^a and θ_T^b respectively.

When a new pair of an initial and a target point $(\boldsymbol{\theta}_I, \boldsymbol{\theta}_T)$ is given, we consider a simple approximation of $f(\boldsymbol{\theta}_I, \boldsymbol{\theta}_T)$ based on a set of learned motor commands $\{\boldsymbol{w}^{ab}\}$ where the set computed for the grids in section V-B,

$$\boldsymbol{w}^{ab} = f(\boldsymbol{\theta}_I^a, \boldsymbol{\theta}_T^b).$$

We first represent each point as the linear combination of the grids as

$$\boldsymbol{\theta}_I = \sum_a \alpha_a \boldsymbol{\theta}_I^a, \quad \alpha_a \geq 0, \quad \sum_a \alpha_a = 1$$
$$\boldsymbol{\theta}_T = \sum_b \beta_b \boldsymbol{\theta}_T^b, \quad \beta_b \geq 0, \quad \sum_b \beta_b = 1$$

We choose $\{\alpha_a\}$ $\{\beta_b\}$ to be the optimal coefficients of the following optimization problems,

$$\max\Big[\sum_a \alpha_a\Big],$$
$$\text{such that} \quad \boldsymbol{\theta}_I = \sum_a \alpha_a \boldsymbol{\theta}_I^a, \quad \alpha_a \geq 0, \quad \sum_a \alpha_a = 1,$$

and

$$\max\Big[\sum_b \beta_b\Big],$$
$$\text{such that} \quad \boldsymbol{\theta}_T = \sum_b \beta_b \boldsymbol{\theta}_T^b, \quad \beta_b \geq 0, \quad \sum_b \beta_b = 1.$$

These problems are solved efficiently with a linear programming (LP). The solution is so sparse that only 3 coefficients of $\{\alpha_a\}$ and $\{\beta_b\}$ are non-zero. Once these coefficients are obtained, the motor command for $\boldsymbol{w}$ is computed as follows

$$\boldsymbol{w} = f(\boldsymbol{\theta}_I, \boldsymbol{\theta}_T) = \sum_{ab} \alpha_a \beta_b f(\boldsymbol{\theta}_I^a, \boldsymbol{\theta}_T^b) = \sum_{ab} \alpha_a \beta_b \boldsymbol{w}^{ab}.$$

Although this is an approximation of the function $f(\boldsymbol{\theta}_I, \boldsymbol{\theta}_T)$, some preliminary numerical results show it works fine.

VI. DISCUSSION AND CONCLUSION

It has been shown that a simple assumption on the preference of the motor command representation solves the motor planning for a reaching task. The resulting motor commands are represented with a set of coefficients of the basis. The coefficients are sparse, and a synergy-like structure is observed. This idea might be useful for robotics to create human-like movements.

We note that we did not assume any noise [3] nor feedback [12]. It is clear that there are noises and a feedback control is necessary for precise control. Our approach can be extended naturally to implement them, and it is one of our future works. Also, biologically plausible basis functions should be considered. Although the arm was restricted to the horizontal plane, we believe extension to three dimensional space with gravity is not difficult.

REFERENCES

[1] T. Flash and N. Hogan, "The co-ordination of arm movements: An experimentally confirmed mathematical model," *J. Neurosci.*, vol. 5, pp. 1688–1703, 1985.
[2] Y. Uno, M. Kawato, and R. Suzuki, "Formation and control of optimal trajectory in human multijoint arm movement. minimum torque-change model," *Biological Cybernetics*, vol. 62, no. 4, pp. 275–288, 1989.
[3] C. M. Harris and D. M. Wolpert, "Signal-dependent noise determines motor planning," *Nature*, vol. 394, pp. 780–784, 1998.
[4] M. Haruno and D. M. Wolpert, "Optimal control of redundant muscles in step-tracking wrist movements," *J. Neurophysiol.*, vol. 94, pp. 4244–4255, 2005.
[5] Y. Sakaguchi and S. Ikeda, "Motor planning and sparse motor command representation," in *Proc. of CNS*2006*, 2006, p. 89.
[6] ——, "Motor planning and sparse motor command representation," *Neurocomputing*, vol. 70, issues 10-12, pp. 1748–1752, 2007.
[7] A. d'Avella, P. Saltiel, and E. Bizzi, "Combinations of muscle synergies in the construction of a natural motor behavior," *Nature Neuroscience*, vol. 6, no. 3, pp. 300–308, 2003.
[8] J. C. Houk, J. T. Buckingham, and A. G. Barto, "Models of the cerebellum and motor learning," *Behavioral and Brain Science*, vol. 19, pp. 368–383, 1996.
[9] B. A. Olshausen and D. J. Field, "Emergence of simple-cell receptive field properties by learning a sparse code for natural images," *Nature*, no. 381, pp. 607–609, 1996.
[10] H. Zou and T. Hastie, "Regularization and variable selection via the elastic net," *J. R. Statistical Society, Series B*, vol. 67, no. 2, pp. 301–320, 2005.
[11] M. Katayama and M. Kawato, "Virtual trajectory and stiffness ellipse during multijoint arm movement predicted by neural inverse models," *Biological Cybernetics*, vol. 69, pp. 353–362, 1993.
[12] W. Li and E. Todorov, "Iterative linear quadratic regulator design for nonlinear biological movement systems," in *Proc. of ICINCO 2004*, 2004, pp. 222–229.
[13] T. Oyama and Y. Uno, "Estimation of a human planned trajectory from a measured trajectory," *Systems and Computers in Japan*, vol. 37, no. 9, pp. 1–11, 2006.
[14] J. Wood, S. Meek, and S. Jacobsen, "Quantitation of human shoulder anatomy for prosthetic arm control-II. anatomy matrices," *J. Biomech.*, vol. 22, no. 4, pp. 309–325, 1989.
[15] D. D. Lee and H. S. Seung, "Learning the parts of objects by non-negative matrix factorization," *Nature*, vol. 401, no. 6755, pp. 788–791, 1999.